INPUT IMPEDANCE OF A SLOTTED CYLINDER ANTENNA

BY

CHARLES ASBURY HOLT III

B.S., United States Military Academy, 1943
M.S., University of Illinois, 1947

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1949

URBANA, ILLINOIS

1949

INPUT IMPEDANCE OF A SLOTTED CYLINDER ANTENNA

BY

CHARLES ASBURY HOLT III

B.S., United States Military Academy, 1943
M.S., University of Illinois, 1947

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1949

URBANA, ILLINOIS

1949

ACKNOWLEDGMENT

For providing the opportunity for the preparation of the thesis the author wishes to express his gratitude to his organization, the United States Air Force, and in particular to the Civilian Institutions Group, Air Force Institute of Technology, Wright-Patterson Air Force Base, Dayton, Ohio. For advice, guidance, and encouragement the author is especially grateful to Doctor E. C. Jordan. Also, special acknowledgment is due Mr. C. C. Nash and his co-workers for the use of their impedance-measuring equipment.

R. UNIV. BIBLIOTHEEK LEIDEN

INPUT IMPEDANCE OF A SLOTTED CYLINDER ANTENNA

The specific problem is the determination of the input impedance of a slot-cylinder antenna in terms of the wave length and the dimensions of the antenna. This antenna consists of a hollow metal cylinder with a thin axial slot connecting the interior and exterior regions. The slot is short-circuited at both ends with the feed voltage applied across the slot at its center-point. The ends of the cylinder may be either open or closed. The slot width is very small compared with the wave length, and the assumption is made that the metal is perfectly conducting.

There are many desirable features of the slot-cylinder antenna. Horizontal polarization is obtained from a slot-cylinder mounted vertically. An approximate circular radiation pattern in the horizontal plane is obtained for antenna diameters less than 0.14 wave length, the pattern becoming more directive in the area adjacent to the slot for larger diameters. Slotted cylinders placed in tandem give a vertical pattern identical with that of a co-linear array. Also, a suitable band width for frequency-modulated broadcasting and television may be obtained.

In order to find the input impedance the slot region is viewed as a transmission line with two thin axial strips, one on each side of the slot, regarded as the energy guides, or current conveyors. These strips have a distributed reactance due to magnetic flux passing through the slot and a distributed admittance due to the conducting cylinder and the slot capacitance. Specifically, the slot voltage is defined as the line integral of the electric field directly across the narrow dimension of the slot; the slot current is defined as the axial current near the slot; and the slot inductance is defined as the magnetic flux passing through the slot per unit length per unit current.

The assumption is made that the axial component of the electric field in the slot region is negligible. Measurements have shown that the axial component of the electric field inside the cylinder is negligible and that horizontal polarization is obtained from a slot-cylinder mounted vertically. The boundary conditions of the theoretical case of an infinitely long slot-cylinder, uniformly fed along the slot with equal in-phase voltages are satisfied by a transverse electric wave. Thus the assumption appears reasonable. From the above definitions and the assumption of negligible axial electric field in the slot region the line equations given below are easily developed.

$dI/dz = -YV$, $dV/dz = -ZI$, where $Y = G + jB$ and $Z = jX$.

The input impedance is that of two similar short-circuited lines in parallel. The line is assumed to be uniform. Approximate average values of the line parameters are determined. From ordinary uniform transmission line theory it follows that the input impedance is $Z_{in} = \frac{1}{2}Z_o \tanh \gamma l$, where l represents one-half the total slot length, Z_o is the characteristic impedance $(\sqrt{Z/Y})$, and γ is the propagation constant $(\gamma = \alpha + j\beta = \sqrt{YZ})$. Some useful relations between the line parameters are: $\alpha = (\frac{1}{2}X)\,(G/\beta) = \sqrt{\frac{1}{2}X}\sqrt{(B^2 + G^2)^{\frac{1}{2}} - B}$, $\beta = (\frac{1}{2}X)\;(G/\alpha) = \sqrt{\frac{1}{2}X}\sqrt{(B^2 + G^2)^{\frac{1}{2}} + B}$, $\beta^2 - \alpha^2 = BX$, and $Z_o = X/(\beta - j\alpha)$. The problem now consists of finding suitable expressions for B, X, and G in terms of the wave length and the dimensions of the antenna.

The slot distributed susceptance B is taken to be the same as that of an infinitely long slot-cylinder of similar transverse dimensions, uniformly fed along the slot with equal in-phase voltages. The applied field is assumed to be uniform across the narrow dimension of the slot. Let (ρ, θ, ψ) represent the cylindrical coordinates and let the cylinder's radius be denoted by a. Then at $\rho = a$, E_θ may be expressed by a Fourier series, E denoting electric intensity. Field expressions satisfying the wave equation and the boundary conditions are readily found. From the fields the surface current densities

may be found. The slot distributed susceptance due to the conducting cylinder is determined by the current densities at the slot. The slot electrostatic capacitance is used to determine the susceptance due to the displacement current across the slot, the inner and outer surfaces of the cylinder acting somewhat like guard plates in reducing edge effect.

Having found the slot distributed susceptance it is possible to determine the slot distributed reactance $X = \omega L$. As the slot width is very small, the contribution of the circumferential current densities to the magnetic flux passing through the slot is negligible. From this it follows that the slot distributed inductance is a function of the slot dimensions only and independent of frequency, as in ordinary line theory. To find a suitable expression for L use is made of the relation $\beta^2 - a^2 = BX$. For frequencies considerably above the cut-off frequency (that is, the frequency at which $B = 0$), then $\beta^2 >> a^2$ and $\beta = (BX)^{\frac{1}{2}}$. B is approximately equal to $b_1\omega + b_2 - b_3/\omega$ where b_n denotes a positive coefficient, and $X = \omega L$. The evaluation of L follows from the fact that β approaches 2π/wave length of free space as ω increases without limit.

The slot distributed conductance G results from equating the radiated power, found from the approximate current density distribution, to the supplied power expressed in terms of G. The circumferential current density is taken as sinusoidal along the axis, this being justified by previous experimental measurements. The beta constant may be determined approximately by neglecting the alpha constant; for frequencies at or below cut-off beta is determined by using the conductance of the corresponding infinite cylinder, the method being similar to the method of finding B. The current density distribution about the circumference is taken to be cosinusoidal, this distribution being made to agree as closely as possible with the circumferential distribution of the infinite cylinder. Schelkunoff's general radiation formula is employed to find the total radiated power.

The power supplied to an axial element of the cylinder is $\frac{1}{2}G\,dz\,V^2(z)$, where $V(z)$ is the slot voltage. Integration over the slot length gives the total power supplied. Equating the power supplied to the power radiated determines G.

As a balanced line was available for impedance measurements, a half cylinder was used, mounted on a ground plate, in the experimental work. The cylinder was divided by a plane containing the axis and bisecting the slot. A 50 ohm coaxial feed line, a Eureka slotted measuring line with a bolometer detector, a standing-wave-amplifier, tuning stubs, and an oscillator (350 to 1116 megacycles) were employed. Measurements were made on a single antenna, but for nine different slot lengths.

The calculated impedance curves compared very favorably with the experimental curves for frequencies below second anti-resonance. At and above second anti-resonance the results were unsatisfactory, probably due primarily to the poor assumption of a cosinusoidal current density distribution about the circumference. Fortunately, the frequency range of greatest interest is below second anti-resonance. Comparison of the analytical method with more experimental results would be desirable.

As a circular radiation pattern in the horizontal plane is usually desirable, a small diameter:wave length ratio is essential. The theory reveals that this ratio may be decreased by capacitive loading of the slot, by decreasing the slot width, or both. However, a sacrifice in band width will result. Increasing the slot length should lower the diameter:wave length ratio at the best operating frequency and also improve the band width. The band width of the experimental antenna, based on a 2:1 reflection standard, was approximately 12 per cent for the longest slot length used.

The slot-cylinder antenna appears very promising for frequency-modulated broadcasting, for television, and for other communication purposes at very-high and ultra-high frequencies. Its usefulness should eventually be greatly extended.

VITA

The author, Charles Asbury Holt III, was born February 22, 1921, at Staunton, Virginia. He attended the United States Military Academy at West Point, New York, from 1939 to 1943, receiving a Bachelor of Science degree in engineering. During the war he served with the United States Air Force as a pilot, was wounded in combat, and hospitalized for a year.

From 1946 to February 1949 the author attended the University of Illinois, receiving the Master of Science degree in electrical engineering in June 1947. He is a member of Sigma Xi and Pi Mu Epsilon mathematical society. He is an officer of the United States Air Force, presently assigned to the Air Material Command at Wright-Patterson Air Force Base, is married, and has one child.

Digitized by the Internet Archive
in 2026

https://archive.org/details/IA41549807_0061